The Four-Step Method

The Four-Step Method is a systematic approach to problem solving that can be used for solving any problem. You may want to discuss these steps with your students before beginning work with the Teaching Problems. Understanding the purpose of each step can help students experience greater success.

Step 1: FIND OUT

The first step for students in solving a problem is to make sure they know what the problem is about and what they are being asked to find. Encourage them to try explaining the problem in their own words; this helps them better understand the information. They should ask themselves:

- What is happening in the problem?
- What do I have to find out to solve the problem?
- Are there any words or ideas I don't understand?
- What information can I use?
- Am I missing any information that I need?

Step 2: CHOOSE STRATEGIES

After students have identified what they are looking for and they know what information they have, they can make a plan for solving the problem. Now is the time to choose the strategy or combination of strategies that they think will be most helpful. They will find that there is often more than one way to solve the problem. In some cases, the problem may have to be broken down into smaller problems before the larger problem can be solved.

Step 3: SOLVE IT

Students now use the strategy they have chosen to solve the problem. It is very important that they record their work in a way that lets them see what they have completed. It is possible they will discover that the strategy they chose is not as helpful as they thought it would be. Emphasize that they should not be discouraged, but rather choose a different strategy and try again.

Step 4: LOOK BACK

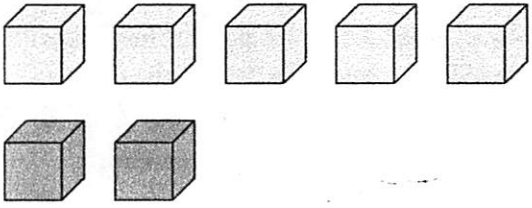
After students have solved the problem, they should always check their answer by reading the problem again, looking back over each step, and checking their calculations. They should ask themselves:

- Did I answer the question asked in the problem?
- Is more than one answer possible?
- Is my math correct?
- Does my answer make sense? Is it reasonable?
- Can I explain why I think my answer is correct?

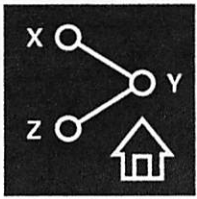
Act Out or Use Objects

Acting out a problem or having objects to move around gives students visual images of the problem and of the steps they must take to solve it. Using this strategy allows students to visualize arrangements, combinations, and relationships in the elements of a problem. Common manipulatives such as Pattern Blocks, cubes, play money, and even scraps of colored paper work well to represent numbers of items and colors.

Example Jessica has 7 balloons. She has 3 more blue balloons than red balloons. How many balloons of each color does Jessica have?



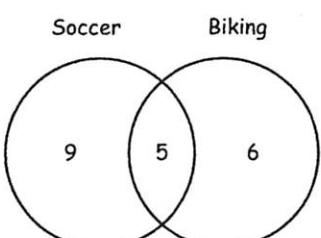
Students can use cubes to represent the balloons. They might start with 7 blue cubes (for the 7 balloons), then exchange one cube at a time for a red cube until they find the combination that gives Jessica 3 more blue than red balloons.



Use or Make a Picture or Diagram

Some problems give a picture, diagram, or map as part of the data. For other problems, it may be helpful for students to draw their own picture or diagram. A simple pictorial representation can often help students understand and work with the data in the problem.

Example On Saturday, 14 scouts in all played soccer and 11 scouts in all went biking. Five of the scouts did both. How many scouts were there altogether?



A Venn diagram would be helpful for solving this problem. After determining the numbers that belong in the three sections of the diagram, students could add those to find there were 20 scouts in all.

Guess and Check

Sometimes it is helpful to guess an answer when trying to solve a problem. When students use this strategy, they guess a number and then check to see if it fits with the other data given in the problem. If it does not, they decide whether their guess was too high or too low, then try to come closer with their next guess. They keep guessing and checking until they find a correct answer.

Example Jerry has 8 more points than Mona. Sam has 4 fewer points than Mona. Together they have 28 points. How many points does each person have?

Guess #1 Mona has 10 points
10 + 8 = 18, so Jerry has 18
10 - 4 = 6, so Sam has 6
10 + 18 + 6 = 34, too high

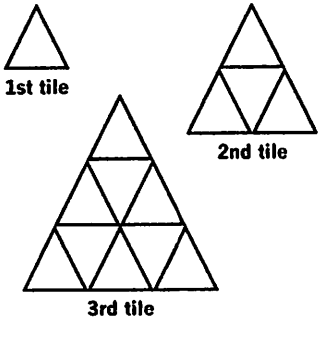
Guess #2 Mona has 8 points
8 + 8 = 16, so Jerry has 16
8 - 4 = 4, so Sam has 4
8 + 16 + 4 = 28

The student's first guess for Mona's points gave a total that was too high. A lower guess leads to the correct total of 28, so the student now knows that Jerry has 16 points, Mona has 8, and Sam has 4.

Use or Look for a Pattern

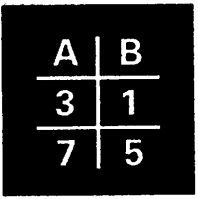
Looking for patterns is a very important strategy for problem solving; it is used to solve many different kinds of problems. A pattern occurs when a relationship is repeated again and again. A pattern may be numerical, visual, or behavioral. Identifying a pattern enables students to predict what will "come next" and what will happen again and again in the same way. In some problems, a pattern is given and students use it to solve the problem. In other problems, students must identify and extend the pattern in order to find a solution. Making a table often reveals patterns in data, so these two strategies are frequently used in combination.

Example Katie is looking for a key that is hidden under a garden tile. She remembers that there are 49 triangles in the tile where the key is hidden. The first 3 tiles she sees are shown here. If the garden tiles keep getting larger in the same way, under what tile should she find the key?



Tiles	Small Triangles
1st	1
2nd	4
3rd	9
4th	16
5th	25
6th	36
7th	49

Placing the information into a table will make it easier for students to see that the number of small triangles increases by odd counting numbers 3, 5, 7, 9, and so on, and that the 7th tile will have 49 small triangles.



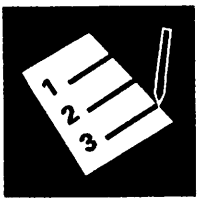
Use or Make a Table

In some problems, students may need to use data from a table or a chart. In other problems, they may need to keep track of data in an orderly way. Making their own tables by listing key numbers in sequence can help students find missing data and discover or extend number patterns. This strategy is often used in combination with other strategies.

Example Devon is 2 years old and Darrin is 8. How old will they be when Darrin is twice as old as Devon?

	Age in Years					
Devon	2	3	4	5	6	
Darrin	8	9	10	11	12	

Creating a table for the problem data helps students see that when Darrin is 12 years old he will be twice as old as Devon, who will be 6 years old.



Make an Organized List

An organized list is a systematic way of recording a series of computations or exploring combinations of items. This strategy helps organize a student's thinking. A list makes it easy to see what has been done and to identify steps that still need to be completed. An organized list is especially helpful when a student wants to consider *all* the possibilities in order to find those that fit the problem.

Example There are more bicycles than cars in the park. There are 20 wheels in all. How many cars and how many bicycles can there be in the park?

Bicycles	Wheels	Cars	Wheels
1	2	1	4
2	4	2	8
3	6	3	12
4	8	4	16
5	10	5	20
6	12	6	24
7	14	7	28
8	16	8	32

After making an organized list of possibilities, students can review them to find the ones that fit the problem ("20 wheels in all"). There are two possible answers: 4 bicycles and 3 cars, or 8 bicycles and 1 car.

Work Backwards

To solve certain problems, it is necessary to begin with data presented at the end of the problem and then work backwards.

Example Kim has 3 more quarters than dimes. She has 1 more dime than nickels. She has 2 nickels. How much money does Kim have?

	2 nickels	$ .10
2 + 1 = 3, so	3 dimes	$ .30
3 + 3 = 6, so	6 quarters	$1.50
		$1.90

Starting with the last information given, 2 nickels, students would work backwards, using the clues to find how many of each type of coin Kim had. They would then add the values to get a total of $1.90.

Use Logical Reasoning

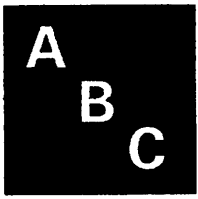
Logical reasoning is actually involved in all problem solving. However, certain problems include or imply conditional statements, such as: "if something is true, then..." or "if something is not true, then..." Problems like these require more formal logical reasoning as students step their way through the clues.

Example Use these clues to match Tanya, David, and Leah with their pets, which are a dog, a lizard, and a gerbil.

- Tanya is allergic to fur.
- David takes his animal for walks.

	Dog	Lizard	Gerbil
Tanya	No	Yes	No
David	Yes	No	No
Leah	No	No	Yes

Using a logic chart, or matrix, will help students analyze the information in each clue. If Tanya is allergic to fur, could she have a dog? A lizard? A gerbil? If Tanya has a lizard, can anyone else have the lizard? Following this line of thinking, students write Yes or No in each cell of the chart until they see that Tanya has a lizard, David has a dog, and Leah has a gerbil.



Make It Simpler

Sometimes a problem can be made simpler by reducing large numbers to small numbers, or by reducing the number of items given in a problem. Having a simpler representation can make it easier to recognize the operation or process that can be used to solve the more complex problem. The simpler representation may even reveal a pattern that can be used to solve the problem.

Example Jay and his friends are having a contest. They want to see who is the best Tic-Tac-Turtle player. The four boys are Jay, Tom, Mark, and Alberto. Only 2 boys at a time play the game. Each boy has to play every other boy one time. How many games will the 4 boys play in all?

2 Boys	3 Boys	4 Boys
Jay and Tom	Jay and Tom	Jay and Tom
	Jay and Mark	Jay and Mark
	Tom and Mark	Jay and Alberto
		Tom and Mark
		Tom and Alberto
		Mark and Alberto

Students can simplify the problem by starting an organized list with 2 boys. They can then extend their list to show how many games 3 boys must play, and finally 4 boys.

Brainstorm

This strategy is often used when all else fails. When students cannot think of a similar problem they have solved before, and they cannot think of another strategy to use, brainstorming is a good strategy to try. It leads students to look at a problem in new and unusual ways. They should be encouraged to open up, stretch, allow for inspiration, be creative, be flexible, and keep on trying until a light goes on!

Example What could this code mean?
100 P i a D

parts pies paints

people in a _____
pounds in a _____

pieces in a _____

pennies in a _____ DOLLAR!

Although this would not be considered a true mathematics problem, and the brainstorming involves language skills, it requires creative thinking about quantitative relationships to figure out that 100 P i a D stands for 100 pennies in a dollar.